EFFICIENCY MEASUREMENT OF DELIVERY POST OFFICES USING FUZZY DATA ENVELOPMENT ANALYSIS (POSSIBILITY APPROACH)

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Abstract: This article integrates fuzzy set theory in Data Envelopment Analysis (DEA) framework to compute technical efficiency scores when input and output data are imprecise. In conventional DEA inputs and outputs data are precise. However, traffic and transportation take place in an uncertain environment and input and output data might be imprecise. This article proposes a possibility approach for solving DEA models with fuzzy data. For the special case in which fuzzy membership functions of fuzzy data are trapezoidal, the possibility approach transforms fuzzy DEA models into linear programming models. By using a group of five post offices in Serbia, this approach is empirically illustrated.

Keywords: fuzzy data envelopment analysis, possibility theory, possibility measure, efficiency.

1. Introduction

Over the past three decades Data Envelopment Analysis (DEA) has emerged as a useful tool for business entities and organizations to evaluate their activities. DEA evaluates the efficiencies of a set of homogenous decision making units (DMUs) with multiple performance criteria. Data used in evaluating the efficiency usually consists of multiple inputs and multiple outputs which represent possibly conflicting criteria. DEA evaluates the relative efficiency of DMUs by using a ratio of the weighted sum of outputs to the weighted sum of inputs with the weights being variable. DEA identifies the source and the amount of inefficiency in each input relative to each output for a target decision making unit based on peer-group comparisons.

There are some limitations of DEA that have to be considered. Because DEA is a methodology focused on frontiers, small changes in data can change efficient frontiers significantly. Therefore, to successfully apply DEA, we have to have accurate measurement of inputs and outputs. However, in some situations it is difficult to measure inputs and outputs in an accurate way to obtain precise data.

In recent years, fuzzy set theory has been proven to be useful tool for imprecise data in DEA models. The DEA models with fuzzy data, called “fuzzy DEA” models, take the form of fuzzy linear programming models (Lertworasirikul, 2003).

There is no universally accepted approach for solving the fuzzy DEA model. In this paper...
we used possibility approach to solve DEA with imprecise data. We measure efficiency of five Serbian post offices considering two inputs (number of employees and average waiting time) and one output (total number of handled items). Average waiting time is fuzzy input in developed model. Membership functions are of triangular shape. Applying the possibility approach we transform Fuzzy DEA model to a linear programming model and solve by LINDO (software tool for solving linear, integer and quadratic optimization models).

2. Literature Review on DEA and Fuzzy DEA

Many applications of DEA can be found in the literature. The method has been used to evaluate the efficiency of DMUs including banking systems, educational institutions and post offices. DEA has also been applied in various modes of transportation.


Papers that have been published on solving fuzzy DEA problems can be categorized following several distinct approaches. Tolerance approach defines tolerance levels on constraint violations. In the defuzzification approach, fuzzy inputs and fuzzy outputs are first defuzzified into crisp values. The resulting crisp model can be solved by an LP solver. In α-level based approach, the fuzzy DEA model is solved by parametric programming using α-cuts. In fuzzy ranking approach both fuzzy inequalities and fuzzy equalities in the fuzzy CCR model are defined by ranking methods so that the resulting model is a bi-level linear programming model. Possibility approach uses concepts of possibility measures and chance-constrained programming to transform a fuzzy DEA model into a well-defined possibility model. Credibility approach uses the “expected credits” of fuzzy variables to deal with uncertainty in fuzzy objectives and fuzzy constraints. The expected credits of fuzzy variables are derived by using credibility measures.

Lertworasirikul et al. (2003) proposed a possibility approach in which fuzzy constraints are treated as fuzzy events and fuzzy DEA model is transformed into possibility DEA model by using possibility measures on fuzzy events. Mugera (2011) applied fuzzy DEA to compute the technical efficiency scores of 34 DMUs using the α-cut level approach. Nedeljkovic and Drenovac (2008) used fuzzy DEA, credibility approach, to measure efficiency of Serbian post offices.

3. Conventional DEA Model

Data Envelopment Analysis (DEA) is a non-parametric methodology for measuring
efficiency within a group of decision-making units (DMUs) that utilize several inputs to produce a set of outputs. DEA models provide efficiency scores that assess the performance of different DMUs in terms of either the use of several inputs or the production of certain outputs. The input-oriented DEA scores vary in $(0, 1]$, the unity value indicating the technically efficient units.

Suppose that there are $n$ decision making units (DMUs), each of which has $m$ inputs and $r$ outputs of the same type. All inputs and outputs are assumed to be nonnegative, but at least one input and one output are positive. The following notation will be used throughout the paper:

- $\text{DMU}_i$ is the $i$th DMU,
- $\text{DMU}_0$ is the target DMU,
- $x_i \in \mathbb{R}^{m \times 1}$ is the column vector of inputs consumed by $\text{DMU}_i$,
- $x_0 \in \mathbb{R}^{m \times 1}$ is the column vector of inputs consumed by the target DMU,
- $X \in \mathbb{R}^{m \times n}$ is the matrix of inputs of all DMUs,
- $y_i \in \mathbb{R}^{r \times 1}$ is the column vector of outputs produced by $\text{DMU}_i$,
- $y_0 \in \mathbb{R}^{r \times 1}$ is the column vector of outputs produced by the target DMU,
- $Y \in \mathbb{R}^{r \times n}$ is the matrix of outputs of all DMUs,
- $u \in \mathbb{R}^{m \times 1}$ is the column vector of input weights,
- $v \in \mathbb{R}^{r \times 1}$ is the column vector of output weights.

For the CCR model, a DMU is inefficient if it is possible to reduce any input without increasing any other inputs and achieve the same levels of output (Cooper et al. 2006). If the efficiency of a DMU is equal to 1, the DMU is weakly efficient (technically efficient).

The CCR model is formulated as the following linear programming model:

\[
\begin{align*}
\text{(CCR)} & \quad \max_{u, v} \quad v^T y_0 \\
\text{S.t.} & \quad u^T x_0 = 1 \\
& \quad -u^T X + v^T Y \leq 0 \\
& \quad u \geq 0 \\
& \quad v \geq 0
\end{align*}
\]

4. Possibility Measure and Fuzzy Event

Similar to the way that measure theory provides the basis for the probability theory, fuzzy sets could be used as a basis for the possibility theory. Also, fuzzy variable is associated with a possibility distribution in the same manner that a random variable is associated with a probability distribution (Lertworasirikul et al. 2003).

Let $(\Theta_i, P(\Theta_i), \pi_i)$, for each $i=1, 2, \ldots, n$, be a possibility space with $\Theta_i$ being the nonempty set of interest, $P(\Theta_i)$ the collection of all subsets of $\Theta_i$, and $\pi_i$ the possibility measure from $P(\Theta_i)$ to $[0, 1]$.

Given a possibility space $(\Theta_i, P(\Theta_i), \pi_i)$ Zadeh (1965) defined a fuzzy variable, $\xi$, as a real valued function defined over $\Theta_i$ with the membership function:
\[ \mu_i(s) = \pi(\{ \theta_i \in \Theta \mid \xi(\theta_i) = s \}) \]
\[ = \sup_{\theta_i \in \Theta_i} \{ \pi(\{ \theta_i \}) \mid \xi(\theta_i) = s \}, \forall s \in \mathbb{R} \]

Let \((\Theta, P(\Theta), \pi)\) be a product possibility space such that \(\Theta = \Theta_1 \times \Theta_2 \times \ldots \times \Theta_n\) and from possibility theory
\[ \pi(A) = \min \{ \pi(A_i) \mid A = A_1 \times A_2 \times \ldots \times A_n, A_i \in P(\Theta_i) \} \]

Let \(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n\) be fuzzy variables and \(f_j: \mathbb{R}^n \to \mathbb{R}\) be real-valued functions, for \(j = 1, \ldots, m\). The possibility of the fuzzy event \(f_j(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) \leq 0, j = 1, \ldots, m\) is given by
\[ \pi(f_j(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) \leq 0, j = 1, \ldots, m) = \sup_{s, s_2, \ldots, s_n \in \mathbb{R}} \{ \min \{ \mu_i(s_i) \} \}
\]

5. Fuzzy DEA

The CCR model with fuzzy coefficients takes the form of fuzzy linear programming model and is given as
\[ (\text{FCCR}) \quad \max \quad v^T \tilde{y}_0 \]
\[ \text{S.t.} \quad u^T \tilde{x}_0 = 1 \]
\[ -u^T \tilde{X} + v^T \tilde{Y} \leq 0 \]
\[ u \geq 0 \]
\[ v \geq 0 \]

where \(\beta\) and \(\alpha_0\) \(\in [0,1]\) are pre-specified acceptable levels of possibility for constraints (1) and (2), respectively, while \(\alpha = [\alpha_1, \ldots, \alpha_n]^T \in [0,1]^n\) is a column vector of pre-specified acceptable levels for the vector of the possibility constraint (3).

The following relations are useful for the solution of the PCCR model:

Let \(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n\) be fuzzy variables with normal and convex membership functions. Let \((\cdot)_L^n\) and \((\cdot)_U^n\) denote the lower and upper bounds of the \(\alpha\)-level set of \(\tilde{a}_i, i = 1, \ldots, n\) (Fig. 1). Then, for any given possibility levels \(\alpha_1, \alpha_2\) and \(\alpha_3\) with \(0 \leq \alpha_1, \alpha_2, \alpha_3 \leq 1\),
\[ \pi(\tilde{a}_1 + \ldots + \tilde{a}_n \leq b) \geq \alpha_i \text{ if and only if } (\tilde{a}_1)_L + \ldots + (\tilde{a}_n)_L \leq b \]
\[ \pi(\tilde{a}_1 + \ldots + \tilde{a}_n \geq b) \geq \alpha_i \text{ if and only if } (\tilde{a}_1)_L \times \ldots \times (\tilde{a}_n)_L \leq b \]
\[ \pi(\tilde{a}_1 + \ldots + \tilde{a}_n = b) \geq \alpha_i \text{ if and only if } (\tilde{a}_1)_L + \ldots + (\tilde{a}_n)_L \leq b \text{ and } (\tilde{a}_1)_U \times \ldots \times (\tilde{a}_n)_U \leq b \]
Given that fuzzy inputs and fuzzy outputs of the PCCR model are normal and convex, it follows that the PCCR model can be solved by considering:

\[
\begin{align*}
\text{(PCCR1)} \quad \max \quad & \tilde{f} \\
\text{S.t.} \quad & (v^T\bar{y}_0)_{\beta^U} \geq \tilde{f} \\
& (u^T\bar{x}_0)_{\alpha^U} \geq 1 \\
& (u^T\bar{x}_0)_{\alpha^L} \leq 1 \\
& (-u^T\bar{X} + v^T\bar{Y} \leq 0)_{\alpha^L} \leq 0 \\
& u \geq 0 \\
& v \geq 0
\end{align*}
\]

For the special case, in which fuzzy inputs and fuzzy outputs are trapezoidal fuzzy numbers \(\tilde{u}_i = ((u_i)^L, (u_i)^{L'}, (u_i)^{U'}, (u_i)^U)\) and \(\tilde{v}_i = ((v_i)^L, (v_i)^{L'}, (v_i)^{U'}, (v_i)^U)\) the PCCR1 model becomes linear programming model:

\[
\begin{align*}
\max \quad & \tilde{f} \\
\text{S.t.} \quad & (1-\beta)(v^T\bar{y}_0)_{\alpha^U} + \beta (v^T\bar{y}_0)_{\alpha^U} \geq \tilde{f} \\
& (1-\alpha_o)(u^T\bar{x}_0)_{\alpha^U} + \alpha_o (u^T\bar{x}_0)_{\alpha^L} \geq 1 \\
& (1-\alpha_o)(u^T\bar{x}_0)_{\alpha^U} + \alpha_o (u^T\bar{x}_0)_{\alpha^L} \leq 1 \\
& (1-\alpha)((-u^T\bar{X})^L + (v^T\bar{Y})^L) + \alpha((-u^T\bar{X})^U + (v^T\bar{Y})^U) \leq 0 \\
& u \geq 0 \\
& v \geq 0
\end{align*}
\]

The objective value \(\tilde{f}\) is used to determine if a target DMU is technically efficient in the possibilistic sense at the specified possibility level. Let \(\alpha\) be the set of \(\beta, \alpha_0, \alpha_1, ..., \alpha_n\). Then, a DMU is \(\alpha\)-possibilistic efficient if its \(\tilde{f}\) value at the \(\alpha\) possibility level is greater than or equal to one; otherwise is possibilistic inefficient.

6. Numerical Example

In this section, Fuzzy DEA is applied to compute the technical efficiency scores of
five post offices (DMUs) in Serbia using the possibility approach. The post offices use two inputs (number of employees and average waiting time) to produce output (total number of handled items). The data were obtained from the Postal Enterprise in Serbia and were also used (Drenovac and Nedeljkovic, 2011) to compute technical efficiencies of the set of Serbian post offices using conventional DEA. In this example, all fuzzy constraints should be satisfied with same possibility level, i.e. $\beta = a_0 = a_1 = \ldots = a_n$.

To illustrate the application of the fuzzy DEA, uncertainty is introduced in the data by representing one input (average waiting time) as symmetric triangular fuzzy number. Membership functions are denoted by $(a,b)$ where $a$ is the center while $b$ is the spread. The data are listed in Table 1.

<table>
<thead>
<tr>
<th>Decision making units</th>
<th>Inputs</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number of employees: $(a,b)$</td>
<td>Total number of handled items $(10^6)$</td>
</tr>
<tr>
<td>DMU I</td>
<td>39 (5, 0.5)</td>
<td>2.824</td>
</tr>
<tr>
<td>DMU II</td>
<td>81 (9.27, 0.6)</td>
<td>3.238</td>
</tr>
<tr>
<td>DMU III</td>
<td>49 (2.63, 0.2)</td>
<td>1.974</td>
</tr>
<tr>
<td>DMU IV</td>
<td>73 (5.57, 0.5)</td>
<td>3.975</td>
</tr>
<tr>
<td>DMU V</td>
<td>58 (3.45, 0.4)</td>
<td>2.792</td>
</tr>
</tbody>
</table>

Table 2

Results of Efficiency Values at Three Possibility Levels

<table>
<thead>
<tr>
<th>Possibility levels</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.23 1.12 1</td>
</tr>
<tr>
<td>0.5</td>
<td>1.05 0.77 0.59</td>
</tr>
<tr>
<td>1</td>
<td>1.17 0.96 0.93</td>
</tr>
<tr>
<td>1</td>
<td>1.42 1.16 1</td>
</tr>
<tr>
<td>1</td>
<td>1.35 1.04 1</td>
</tr>
</tbody>
</table>

From Table 2, DMUs 1, 4 and 5 are possibilistically efficient at all possibility levels, while DMUs 2 and 3 are possibilistically efficient only at possibility level equal to 0. The results show that when the possibility level is increased, the efficiency value is decreased.

7. Conclusion

This paper has utilized a possibility approach as a way to solve fuzzy DEA models. The approach transforms a fuzzy DEA model into a possibility DEA model. A numerical example is given to illustrate the implementation of the approach. Efficiencies of five Serbian post offices are measured, considering two inputs (one is fuzzy) and one output. Membership functions of fuzzy input are of triangular shape. In that case the possibility DEA model has the form of a linear programming model. The model is solved by LINDO solver. A possible topic for further research would be to apply other approaches for solving fuzzy DEA models and to compare the results.
References


ODREĐIVANJE EFKASNOSTI POŠTANSKIH JEDINICA PRIMENOM FAZI ANALIZE OBAVIJANJA PODATAKA (PRISTUP ZASNOVAN NA MERI MOGUĆNOSTI)

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Ključne reči: fazi analiza obavijanja podataka, teorija mogućnosti, mera mogućnosti, efikasnost.